

# On the Zero Divisor Graph of the Ring of Integer Modulo $n$

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## Abstract

For a commutative ring  $R$  with non-zero zero divisor set  $Z^*(R)$ , the zero divisor graph of  $R$  is  $\Gamma(R)$  with vertex set  $Z^*(R)$ , where two distinct vertices  $x$  and  $y$  are adjacent if and only if  $xy = 0$ . The zero divisor graph structure of  $\mathbb{Z}_{p^n}$  is described. We determine the clique number, degree of the vertices, size, metric dimension, upper dimension, automorphism group, Wiener index associated to the zero divisor graph of  $\mathbb{Z}_{p^n}$ . Further, we provide a partition of the vertex set of  $\Gamma(\mathbb{Z}_{p^n})$  into distance similar equivalence classes and we show that in this graph the upper dimension equals the metric dimension. Also, we discuss similar properties of the compressed zero divisor graph.

**Keywords:** Ring, zero divisor; zero divisor graph, metric dimension, upper dimension

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